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COMMON FIXED POINTS IN HILBERT SPACE

Seema Sinha*[1], Premlata Verma[2] and G.S.Sao[3]
*[1,3]Dept.of Mathematics, Government ERR PG Science College, Bilaspur(C.G.)
[2] Dept.of Mathematics, Government Bilasa Girls PG College, Bilaspur(C.G.)

ABSTRACT
In this paper we will prove a common fixed point theorem using contraction and rational inequality in Hilbert Space, So the purpose of this paper is establish the generalisation of contraction in Hilbert Space.

Keywords- *Hilbert Space, Common Fixed Point, Parallelogram Law.*

I. INTRODUCTION
In recent years some fixed points of various type of contraction mapping in Hilbert space and Banach spaces were obtained, among others by Browder [1], Browder and Petryshyn[2],Hicks ,Huffman[3],Junck[4], Mujahid Abbas, Miko Jovanovic , Stojan Radenovic , Aleksandra Sretenovic Suzana Simic[5] and Yadav, Hema, Sayyed, S.A. and Badshah, V.H[10].

II. PRELIMINARIES
2.1 NORM : A norm on X is a real-valued function $\|.\| : X \to R$ defined on X such that for any $x, y \in X$ and for all $\lambda \in K$

(a) $\|x\| = 0$ if and only if $x = 0$
(b) $\|x+y\| \leq \|x\| + \|y\|$
(c) $\|\lambda x\| = |\lambda| \, \|x\|$

2.2 NORMED LINEAR SPACE : It is a pair $(X, \|.\|)$ consisting of a linear space X and a norm $\|.\|$. We shall abbreviate normed linear space as nls.

2.3 CAUCHY SEQUENCE : A Sequence $\{x_n\}$ in a normed linear space X is a Cauchy sequence if for any given $\varepsilon > 0$, there exist $n_0 \in N$ such that $\|x_m - x_n\| < \varepsilon$ for $m, n \geq n_0$.

2.4 CONVERGENCE CONDITION IN NLS : A sequence $\{x_n\}$ in a nls X is said to be Convergent to $x \in X$ if for any given $\varepsilon > 0, \exists$ $n_0 \in N$ such that $\|x_n - x\| < \varepsilon$ for $n \geq n_0$

2.5 COMPLETENESS : A nls X is said to be complete if for every Cauchy Sequence in X converges to an element of X.

2.6 BANACH SPACE : A Banach Space $(X, \|.\|)$ is a complete nls.

2.7 INNER PRODUCT SPACE : Let X be a linear space over the scalar field C of complex numbers. An inner product on X is a function $(.\,,.) : XxX \to C$ which satisfies the following conditions

(a) $(x, y) = \overline{(y, x)}$ for $x, y \in X$

(b) $(\lambda x + \mu y, z) = \lambda (x, z) + \mu (y, z)$ for $\lambda, \mu \in C, x, y, z \in X$

(c) $(x, x) \geq 0$; $x\ x) = 0$ iff $x = 0$

2.8 **LAW OF PARALLELOGRAM:**If x and y are any two elements of an inner product space X then $\|x + y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$

or $\|x + y\|^2 \leq 2\|x\|^2 + 2\|y\|^2$

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2.9 **HILBERT SPACE** : An infinite dimensional inner product space which is complete for the norm induced by the inner product is called Hilbert Space.

III. MATERIAL AND METHODS

3.1 THEOREM : If T be the self map satisfying following then we have

$$||Tx-Ty||^2 \leq k \min [||y-Ty||^2, \frac{1}{2}(||x-Ty||^2+||y-Tx||^2), \frac{1}{4}(||x-Tx||^2+||y-Ty||^2),$$

$$\frac{||y-Ty||^2\{1+|x-Tx||^2\}}{1+||x-y||^2}, \frac{||x-Tx||^2\{1+||Tx-Ty||^2\}}{1+||y-Ty||^2},$$

$$\frac{||x-Ty||^2\{1+||x-Tx||^2\}}{1+||x-y||^2}, \frac{||y-Tx||^2\{1+||y-Ty||^2\}}{1+||Tx-Ty||^2}]$$

Then T has fixed point when $0\leq k<1$

Suppose $x=x_{2n}$, $y=x_{2n+1}$ and $Tx_{2n} = Tx_{2n+1}$ then we have

$$||x_{2n+1}-x_{2n+2}||^2 \leq k \min [||x_{2n+1}-x_{2n+2}||^2, \frac{1}{2}(||x_{2n}-x_{2n+2}||^2+||x_{2n+1}-x_{2n+1}||^2),$$

$$\frac{1}{4}(||x_{2n}-x_{2n+1}||^2+||x_{2n+1}-x_{2n+2}||^2), ||x_{2n+1}-x_{2n+2}||^2, ||x_{2n}-x_{2n+2}||^2]$$

$$||x_{2n+1}-x_{2n+2}||^2 \leq k \min [||x_{2n+1}-x_{2n+2}||^2, \frac{1}{2}(2||x_{2n}-x_{2n+1}||^2+2||x_{2n+1}-x_{2n+2}||^2),$$

$$\frac{1}{4}(||x_{2n}-\mathbf{x_{2n+1}}||^2+||\mathbf{x_{2n+1}}-\mathbf{x_{2n+2}}||^2), ||x_{2n+1}-x_{2n+2}||^2, 2||x_{2n}-x_{2n+1}||^2+2||x_{2n+1}-x_{2n+2}||^2]$$

$\leq k\,||x_{2n} - x_{2n+1}||^2$ if $0\leq k<1$

$\leq k^2\,||x_{2n-1} - x_{2n}||^2$

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$\leq k^n\,||x_n - x_{n+1}||^2$

$\rightarrow 0$ as $n \rightarrow \infty$

IV. RESULT AND DISCUSSION

Above shows that $\{Tx_n\}$ is a Cauchy Sequence in H as H is a Hilbert Space and T is self map then Tx_n converges to some point p.

V. CONCLUSION

In this paper, we have proved the existence of a fixed point of T and contraction of T in a Hilbert Space which is unique.

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